Families Ask

Take-Home Page

Families often ask a question like this:

How is "showing your strategy" different from "showing your work"?

"Show your work" is something that many of us have been asked to do on mathematics homework, starting in elementary school and continuing on through high school and college. Here is a familiar example.

$$\frac{x}{40}=\frac{15}{60}$$

Solve for x. Show your work.

To many, "show your work" is the same as "show each step." These steps, explained to the student before working on the problem, might include the following:

Copy the original problem:	$\frac{x}{40}=\frac{15}{60}$
Cross multiply:	$60x = (40)(15)$
Multiply the two given numbers:	$60x = 600$
Divide both sides by 60:	$\frac{60x}{60}=\frac{600}{60}$
Simplify:	$x = 10$

(Note: The phrases were simply added for clarification, since the student would usually just do the steps without an explanation.) This example of solving a proportion may be familiar to you, since it is the standard process for such a problem. Therefore, what is the purpose of showing one's work? There are three possible answers: (1) if a student makes a mistake, the teacher can see where it occurred; (2) it helps the student keep track of where he or she is in solving the problem; and (3) it ensures that the student actually does the work.

If your child is being asked to "show your thinking," then the solution process is a personal choice. The student would solve the problem and explain how it was done. Here are some possible ways that a student might explain his or her thinking about the problem: "I know that 15 is 1/4 of 60, so I just found 1/4 of 40, which is 10"; "I simplified the fraction 15/60 to be 1/4, and then it was easy to see that the numerator for the fraction with 40 in the denominator was 10"; "I looked at the denominators and saw that to get from 40 to 60, I would need to multiply by 1.5. So, in thinking backward, I needed a number to multiply by 1.5 to get 15, which is 10"; and "I compared the numerator of 15 with its denominator 60 and saw that it takes 4 fifteens to make 60, so I needed a 10 in the other numerator because 4 tens is 40."

There are probably other ways a student might solve the proportion, especially because the numbers are considered to be "friendly"; in other words, they are easy to compute. Even if they were not friendly, however, a student could use the strategies mentioned above, such as comparing the numerator to its denominator and making sure the ratio is the same. Such strategies require that a student understands the relationships in this problem (between numerator and denominator, between denominators and equivalent fractions). Why would a teacher ask a student to explain his or her thinking? One reason might be to see whether a strategy is in use that is efficient and will always work. Another reason might be for other students to hear different approaches to the problem. In addition, teachers may want to see not only if a student can find an answer, but also if he or she understands the important proportional relationships in the task.

Showing steps and explaining one's thinking both have a place in a mathematics class. Showing one's steps is a well-established practice, but explaining one's thinking is an important part of doing mathematics that should be included when learning any math concept. Explaining one's thinking has the following benefits:

- A student uses a strategy that makes sense to him or her.
- A student learns from hearing and/or watching other students' strategies.
- There are often many ways to approach a mathematics problem, so the process captures the nature of real mathematics.
- A student is in the role of mathematician rather than observer.
- This practice accommodates different learning styles and different backgrounds.
- A student is not imitating (e.g., a natural teacher's approach), but *inventing*. This higher-level thinking process involves truly "doing mathematics."

In the problem posed above, using the standard process of cross multiplication is more cumbersome than exploring some mental strategies. If students are simply asked to show their work (to follow a standard procedure), they will likely overlook opportunities to use more efficient approaches. When students are encouraged to explain their thinking, they can examine the problem first to decide how they want to solve it.

This process carries over to homework problems. As you work with your child in helping with homework, you can ask, "Let's look at this problem and see whether you can come up with a way to solve it." You may find that your child is much better at thinking up a good strategy than trying to recall someone else's approach to doing the problem.

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